# MODE IDENTIFICATION FROM LINE PROFILES USING THE DIRECT FITTING TECHNIQUE

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**Abstract.** The method of moments and the direct fitting method are the only spectroscopic methods of mode identification which allow a determination of all pulsational parameters. The pulsation parameters are required to predict the light amplitude and phase which can be important discriminants in mode identification. The direct fitting method has several advantages over the method of moments. It is not restricted to low spherical harmonic degree or form of the eigenfunction and is not sensitive to the placement of the continuum. In the last few years the method has been applied to several different types of stars. We briefly describe the method and give some examples of its application.

**Keywords:** line: profiles; stars: oscillations

## 1. Introduction

One way of obtaining confidence that a particular spectroscopic mode identification is correct is to calculate the amplitude and phase of the light variation using the parameters obtained from line profile fitting. The parameters are as follows: the vertical velocity amplitude, $V_p$ and phase, $\phi_p$; the ratio of relative temperature to relative radius variation, $f$, and its phase, $\psi$, and the angle of inclination, $i$. For computational purposes, it is more convenient to use the pseudo velocity and phase, $V_f$, $\phi_f$, rather than the theoretical quantities $f$ and $\psi$. The relationship between these quantities can be found in Balona (2000). We assume that the projected rotational velocity, $v \sin i$, the intrinsic line profile and the limb darkening coefficients are known. We also need to know how the line strength depends on temperature and gravity.

There are only two methods which allow determination of all pulsational parameters – the method of moments (Balona, 1987; Aerts et al., 1992) and the direct fitting technique (Balona and Kambe, 1999). The direct fitting technique has several advantages over the method of moments. Firstly, it is not limited to low values of $\ell$ or the particular form of the eigenfunction. Secondly, it is less susceptible to how the continuum is positioned, since the fitting may be restricted to a region near the line core. The drawback is that it is far more computationally intensive, a problem which is no longer as restrictive as it used to be. In applying the method to multiperiodic stars, the assumption is made that the line profile variations from other modes cancel out within a phase bin. This restriction is closely related to

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the sampling and aliasing problems associated with time series analysis and is not unique to the direct fitting method.

In this article, we report on the application of the method to four very different types of stars. I do not believe that it is necessary to use as many different methods as possible in mode identification. Rather, I believe that it is important to use the *best possible method* suited to the data. If the data is limited, then rough estimates of $\ell$ and $m$ are all that can be deduced. Given enough data, however, a more rigorous method such as the moment or direct fitting methods is to be preferred so that the results may be checked via the photometry.

## 2. An Outline of the Method

To apply the direct fitting method, one needs to compute line profile variations for a particular $(\ell, m)$ given the pulsational parameters. Because there is a large number of parameters, it is important to fix as many of them as possible. If the star is slowly rotating, then temperature perturbations have a negligible effect on the line profile variations, so that $V_f$ and $\phi_f$ can be neglected. These parameters are important in rapidly rotating stars and may, in fact, have a much larger effect than the pulsational velocity, $V_p$. In this case it may be possible to fix the ratio $V_f/V_p$ by making use of theoretical values of $f$ and $\psi$ (e.g. Balona et al., 2002).

The observed line profiles are binned and averaged in a number of phase bins (5 to 10 bins are normally used) for the period under consideration. The line profile for a given $(\ell, m)$ is calculated for these phases and a goodness of fit criterion, $\sigma$, is determined. One or more of the pulsational parameters is then adjusted and the process repeated. One needs to be careful not to locate a local minimum of $\sigma$. To avoid this, a coarse grid of $\sigma$ values is calculated over all physically possible values of the pulsational parameters. Having obtained the approximate global minimum, accurate values of the pulsational parameters are then calculated using one of many techniques, such as the method of steepest descent.

Once the best parameters for suitable ranges of $\ell$ and $m$ have been found, together with their associated goodness of fit, it is important to calculate the predicted light amplitude and phase in a given passband. This cannot be done unless one knows the values of $f$ and $\psi$ from models. In principle, $f$ and $\psi$ could be determined empirically. Except in rapidly rotating stars, the values obtained have such large error margins as to render them valueless. Assuming, however, that $f$ and $\psi$ are known (even approximately), comparison of the predicted to observed light amplitudes and phases can be crucial in selecting which of several solutions with nearly the same goodness of fit is likely to be correct.

## 3. Examples

### 3.1. 19 Mon

The simplest example is a star with a single period and a mode of low degree. For this purpose, we consider the application to the rapidly rotating star B1V star 19 Mon (Balona et al., 2002). Although three periods have been detected in this star, one of them has by far the highest amplitude, so the star can be considered as monoperiodic. In this $\beta$ Cep star values of $f$ and $\psi$ were available from models using Dziembowski's `NADROT` code, so that $V_f/V_p$ could be treated as a fixed parameter. Furthermore, the large value of $v \sin i = 274$ km s$^{-1}$ could be used to constrain the inclination angle $i \approx 90°$. The free parameters are $V_p$, $\phi_p$ and $\phi_f$.

The results showed that the most likely mode is (2, –2), but (3, –3) and (1, –1) gave values of $\sigma$ which were almost as small. However, calculations of the light amplitude and phase are more consistent with (2, –2), giving a good level of confidence to this identification. One of the difficulties here is that the angular velocity of rotation, $\Omega$, is not well known. This means that there is considerable uncertainty in determining the period for zero rotation, which is required for direct comparison with the models. Moreover, the eigenfunction is likely to depart significantly from a spherical harmonic because the ratio of rotation to pulsation frequency is not very small.

### 3.2. 1 Mon

The $\delta$ Scuti star 1 Mon is of particular interest because it has a frequency triplet with practically equal frequency spacing. It is very tempting to regard this as an example of a rotationally split $\ell = 1$ mode. However, many years ago, Balona and Stobie (1980) identified the central component as a *radial* mode on the basis of photometric mode identification. Balona et al. (2001) obtained simultaneous spectroscopy and photometry of the star. Application of the direct fitting technique together with photometric mode identification confirmed that the central component is indeed a radial mode. One of the other components is identified as (1, –1), but the third member of the triplet was not seen, presumably because the amplitude had decreased in the 20 years since it was last observed.

In this case, rotation is very small so that $V_f$ and $\phi_f$ can be neglected. The free parameters are $V_p$, $\phi_p$ and $i$. Results did not produce a unique solution for the central component, $\nu_1$. The identifications (0, 0), (1, 0) and (1, –1) give almost identical values of $\sigma$. However, the photometry clearly supports the (0,0) mode, illustrating the value of combining both spectroscopy and photometry in mode identification.

### 3.3. $\zeta$ Oph

$\zeta$ Oph is a rapidly rotating O9V star in which features moving from blue to red cross the line profile, indicating prograde modes of high degree. Balona and Kambe (1999) identified two frequencies and applied the direct fitting method to obtain $\ell = 8$ and $\ell = 4$. An interesting aspect of this analysis is that the line profile variations are caused mainly by the temperature perturbation and not by the pulsational velocity (i.e. $V_f >> V_p$). This may be true for all high degree modes in rapidly rotating stars. The light variations are not detected, presumably because $\ell$ is too large.

Another interesting aspect is that, whereas the observations clearly rule out retrograde modes, there is little discrimination between (8, –8), (8, –7), (8, –6) in the one case or (4, -4), (4, –3) (4, –2) in the other case. The true identification need not necessarily be the pure sectorial modes (8, –8) and (4, –4). It seems that for modes of high order it becomes increasingly difficult to discriminate between neighbouring values of $m$. This is very unfortunate because the exact value of $(\ell, m)$ as well as the rotational frequency, $\Omega$, is necessary for asteroseismology.

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